Basic of Mathematics

Questions

Q1 - 24 June - Shift 2

Let $x, y > 0$. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(A) 30 (B) 32

(C) 36 (D) 40

Q2 - 26 June - Shift 2

Let p and q be two real numbers such that $p + q = 3$ and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$ is equal to _____.

Q1 - 28 July - Shift 2

Let $S=\left\{x \in [-6,3]-\{-2,2\} : \frac{|x+3|-1}{|x|-2} \geq 0\right\}$ and $T=\{x \in Z : x^2-7|x|+9 \leq 0\}$. Then the number of elements in $S \cap T$ is

(A) 7 (B) 5

(C) 4 (D) 3

Answer Key

June 2022

Q1 (D) Q2 (4)

July 2022

Q1 (D)

Q1 (D)

Using AM ≥ GM

$$\frac{x+x+x+y+y}{5} \geq \left(x^3.y^2\right)^{\frac{1}{5}}$$

$$\frac{3x+2y}{5} \geq \left(2^{15}\right)^{\frac{1}{5}}$$

$$(3x+2y)_{min} = 40$$

Q2 (4)

$p + q = 3 \quad p^4 + q^4 = 369$

$\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}$

$(p+q)^2 = 9$

$p^2 + q^2 = 9 - 2pq$

$$\frac{1}{\left(\frac{1}{p}+\frac{1}{q}\right)^2} = \frac{(qp)^2}{(q+p)^2} = \frac{(qp)^2}{9}$$

$p^4 + q^4 = (p^2+q^2)^2 - 2p^2q^2$

$369 = (9-2pq)^2 - 2(pq)^2$

$369 = 81 + 4p^2q^2 - 36pq - 2p^2q^2$

$288 = 2p^2q^2 - 36pq$

$144 = p^2q^2 - 18pq$

$(pq)^2 - 2 \times 9 \times pq + 9^2 = 144 + 9^2$

$(pq-9)^2 = 225$

$pq - 9 = \pm 15$

$pq = \pm 15 + 9$

$pq = 24, -6$

(24 is rejected because $p^2 + q^2 = 9 - 2pq$ is negative)

$$\frac{(qp)^2}{9} = \frac{16 \times 16}{9} = 4$$